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ON STRONG LIFTING COMPACTNESS FOR THE WEAK* TOPOLOGY

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Dedicated to Prof. B. Volkmann on the occasion of his 60th birthday

Introduction. The notion of lifting compactness was introduced in [**3**], that of strong lifting compactness in [**1**], both for completely regular Hausdorff spaces. It turned out in [**1**] that the strong lifting compactness of a Banach space X under its weak topology is equivalent with each of the following strong properties.

(SL) For every Baire measure μ on X any lifting of $\mathfrak{L}^\infty(\mu)$ is almost strong, respectively there exists an almost strong lifting for $\mathfrak{L}^\infty(\mu)$.

(SB) Every scalarly measurable function from a complete probability space into X is Bochner measurable.

It is therefore natural to ask whether these equivalences hold also for other locally convex topologies. In this paper, we check the weak* topology on conjugates of Banach spaces. In Theorem 3.4 we give a characterization of such conjugate Banach spaces which satisfy condition (SB), from which it becomes obvious that condition (SB) is neither equivalent with condition (SL) nor with strong lifting compactness of the conjugate under its weak* topology (see also the examples in 3.8). We call Banach spaces which satisfy the equivalent conditions of Theorem 3.4 SB*-spaces. These spaces form a strong counterpart to the well-known class of Asplund spaces, since they are definable by a strict equivalence instead of a weak equivalence for measurable functions which characterizes Asplund spaces in the sense of [**20**, **18**]. As a preparation, this weak equivalence characterization for Asplund spaces is derived in Theorem 2.4, and at the same time an equivalent lifting invariance condition. We also introduce L*-spaces, M*-spaces, and (strict) W*-spaces which are related to SB*-spaces and to Banach spaces whose conjugate has the weak Radon Nikodym property of [**17**]. Weak* strongly lifting compact spaces, i.e., such

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