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SPACES ON WHICH UNCONDITIONALLY CONVERGING OPERATORS ARE WEAKLY COMPLETELY CONTINUOUS

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ABSTRACT. Let Ω be a compact Hausdorff space, and let E be a Banach space with unconditional reflexive decomposition, then every unconditionally converging operator T on $C(\Omega, E)$, the space of E-valued continuous functions on Ω, is weakly completely continuous, i.e., T sends weakly Cauchy sequences into sequences that converge weakly.

Introduction. Let $T : X \to Y$ be a bounded linear operator from a Banach space X into a Banach space Y. We say that T is *weakly compact* (w.c.) if for every bounded sequence (x_n) in X, there is a subsequence (x_{n_k}) such that (Tx_{n_k}) converges weakly in Y. We say that T is *weakly completely continuous* (w.c.c.) (also called Dieudonné operator) if for every weakly Cauchy sequence (x_n) in X, the sequence (Tx_n) converges weakly in Y, and we say that T is *unconditionally converging* (u.c.) if for every weakly unconditionally Cauchy series (w.u.c.) $\sum_n x_n$ in X, the series $\sum_n Tx_n$ converges unconditionally in Y. Here recall that a series $\sum_n x_n$ is weakly unconditionally Cauchy if for each x^* in X^* the series $\sum_n |x^*(x_n)|$ is convergent. It is clear that T weakly compact implies T weakly completely continuous which in turn implies T unconditionally converging. In his fundamental paper [**9**] A. Pelczynski looked at spaces on which every unconditionally converging operator is weakly compact. Such spaces are said to have Pelczynski's property (V). In [**9**] Pelczynski showed that among classical Banach spaces, the spaces $C(\Omega)$ of scalar-valued continuous functions on a compact Hausdorff space Ω have property (V), and in [**7**] W. Johnson and M. Zippin showed that more generally any Banach space whose dual is isometric to an L^1 space have property (V). Also in [**9**] spaces with property (u) were introduced; for this recall that a Banach space E has *property* (u) if for any weakly Cauchy sequence (e_n) in E there

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