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A CONGRUENCE FOR $c\phi_{h,k}(n)$

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ABSTRACT. This paper is a sequel to a recent paper [**2**] on congruences for generalized Frobenius partitions. With the aid of some congruence properties for compositions, we will derive a congruence, modulo h^2, for $c\phi_{h,k}(n)$, the number of generalized Frobenius partitions of n with h colors and at most k repetitions, provided $(h, k+1) = 1$.

Introduction. Let $c\phi_{h,k}(n)$ be the number of generalized Frobenius partitions, F-partitions for short, of n with h colors and (at most) k repetitions as introduced in [**3**]. These combinatorial objects are an extension of two classes of F-partitions introduced by Andrews [**1**]. In two recent papers [**3, 4**] the generating functions and the Hardy-Ramanujan-Rademacher expansions for $c\phi_{h,k}(n)$ were derived. In this paper we will prove two congruences for $c\phi_{h,k}(n)$ which are similar to congruences for two other classes of F-partitions.

It has been shown [**2**] that $\sum_{d|(h,n)} \mu(d) c\phi_{h/d}(n/d) \equiv 0 (\operatorname{mod} h^2)$ and $\sum_{d|(h,n)} \mu(d) k\phi_{h/d}(n/d) \equiv 0 (\operatorname{mod} h^2)$ where $c\phi_h(n)(k\phi_h(n))$ are the number of F-partitions of n with h colors without (with unrestricted) repetitions. In this paper we will prove the following.

Theorem 1.

$$\sum_{d|(h,n)} \mu(d) c\phi_{h/d,k} \left(\frac{n}{d}\right) \equiv 0 (\operatorname{mod} h)$$

Theorem 2.

$$\sum_{d|(h,n)} \mu(d) c\phi_{h/d,k} \left(\frac{n}{d}\right) \equiv 0 (\operatorname{mod} hH)$$

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