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NONSELF-ADJOINT DIFFERENTIAL OPERATORS IN DIRECT SUM SPACES

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1. Introduction. In [**8**] Everitt and Zettl considered the problem of characterizing all the self-adjoint operators which can be generated by formally symmetric Sturm-Liouville differential expressions M_p ($p = 1, 2$) defined on two intervals I_p ($p = 1, 2$) with boundary conditions at the endpoints. Their work was motivated by Sturm-Liouville problems which occur in the literature in which the coefficients have a singularity in the interior of the underlying interval. An interesting feature of their work is the possibility of generating self-adjoint operators in this way which are not expressible as the direct sums of self-adjoint operators defined in the separate intervals.

Our objective in this paper is to extend the results of Everitt and Zettl in [**8**] to the case where the differential expressions M_p are arbitrary and there is any finite number of intervals I_p, $p = 1, \dots, N$.

The operators involved are no longer symmetric but direct sums

$$T_0(M) = \bigoplus_{p=1}^{N} T_0(M_p), T_0(M^+) = \bigoplus_{p=1}^{N} T_0(M_p^+),$$

where $T_0(M_p)$ is the minimal operator generated by M_p in I_p and M_p^+ denotes the formal adjoint of M_p, which form an adjoint pair of closed operators in $\oplus_{p=1}^{N} L^2_{w_p}(I_p)$. This fact allows us to use the abstract theory developed in [**1**] for the operators which are regularly solvable with respect to $T_0(M)$ and $T_0(M^+)$. Such an operator S satisfies $T_0(M) \subset S \subset [T_0(M^+)]^*$ and for some $\lambda \in \mathbf{C}$, $(S - \lambda I)$ is Fredholm with zero index. This class of operators is the counterpart of the class of maximal symmetric and self-adjoint operators in the case when $T_0(M)$ is symmetric. Using ideas and results from [**2**], we are also able to characterize all the operators which are regularly solvable with respect to $T_0(M)$ and $T_0(M^+)$ in terms of the $L^2_{w_p}(I_p)$ solutions of

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