ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 22, Number 3, Summer 1992

SUBORDINATION FAMILIES

D.J. HALLENBECK

ABSTRACT. Let $s(F)$ denote the set of functions subordinate to a function F analytic in the unit disk Δ. Let $Hs(F)$ denote the closed convex hull of $s(F)$ and Λ denote the set of probability measures on $\partial\Delta$. Let $R = \{F : F \text{ is analytic in } \Delta \text{ and } Hs(F) = \{\int_{|x|=1} F(xz)\,d\mu(x) : \mu \in \Lambda\}\}$. Let R_Σ denote those functions analytic in Δ such that the set of support points of $s(F)$ is $\{F(xz) : |x| = 1\}$. In this paper we investigate R and R_Σ. We prove that if F is a univalent function in R and $(F(z) - F(0))/F'(0)$ has positive Hayman index, then F is in R_Σ.

Introduction. Let $\Delta = \{z : |z| < 1\}$ and let A denote the set of functions analytic in Δ. If $f \in A$, $f(z) = \sum_{n=0}^{\infty} a_n z^n (z \in \Delta)$, then let $\bar{m}(r, f) = \sum_{n=0}^{\infty} |a_n| r^n$. A is a locally convex linear topological space with respect to the topology given by uniform convergence on compact subsets of Δ. Let $B_0 = \{\phi : \phi \in A \text{ and } |\phi(z)| \leq |z|\}$. A function f is called a support point of a compact subset $\mathcal{F}$ of A if $f \in \mathcal{F}$ and if there is a continuous, linear functional J on A so that $\operatorname{Re} J(f) = \max\{\operatorname{Re} J(g) : g \in \mathcal{F}\}$ and $\operatorname{Re} J$ is not constant on $\mathcal{F}$. We denote the set of support points of $\mathcal{F}$ by $\Sigma\mathcal{F}$, the set of extreme points of $\mathcal{F}$ by $E\mathcal{F}$ and the closed convex hull of $\mathcal{F}$ by $H\mathcal{F}$.

Let $D = \{f : f \in A \text{ and } \int_\Delta \int |f'(z)|^2\,dy\,dx < +\infty \text{ where } z = x + iy\}$ and $S = \{f \in A : f(0) = 0, f'(0) = 1 \text{ and } f \text{ is univalent}\}$. If we set $M(r, f) = \max_{|z| \leq r} |f(z)|$ then $\alpha = \lim_{r\to 1^-} (1 - r)^2 M(r, f) \leq 1$ exists for each $f \in S$ and is called the Hayman index of f [**9**, p. 163, **17**, p. 141]. It is known that f has a unique direction of maximal growth $e^{i\theta_0}$ if $\alpha > 0$, in the sense that $\lim_{r\to 1^-} (1 - r)^2 |f(re^{i\theta_0})| = \alpha$ and $\lim_{r\to 1^-} (1 - r)^2 |f(re^{i\theta})| = 0$ for $\theta \neq \theta_0$ [**9**]. Let $S_\alpha = \{f : f \in S \text{ and } \alpha > 0\}$.

Received by the editors on October 4, 1988 and in revised form on April 16, 1989.
1980 AMS *Mathematics Subject Classification.* Primary 30C80; Secondary 30C55.
Key words and phrases. Support points, subordination.