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NONHOMOGENEITY OF POWERS OF COR IMAGES

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ABSTRACT. A space is called a Cor image if it is a Hausdorff continuous image of some compact ordered space. A space is called homogeneous if any point can be mapped to any other point by some autohomeomorphism of the space. By investigating special kinds of points, we supply necessary conditions for some power of a compact space to be homogeneous. Applying this, we prove that if some power of a Cor image is homogeneous, then the Cor image must be first countable.

1. Introduction. A space is homogeneous if any point can be mapped to any other point by an autohomeomorphism of the space. We call a space powerhomogeneous if some power of the space is homogeneous. This paper is devoted to a partial result on the basic problem: Which compact spaces are powerhomogeneous? Of course, if X is homogeneous, then all powers of X are homogeneous. A convergent sequence with limit point is a simple example of a nonhomogeneous space X such that X^ω is homogeneous. A famous connected example is the closed unit interval I. Keller [**3**] has shown that the Hilbert cube is homogeneous. A simple example of a compact space X that is not powerhomogeneous, due to van Douwen [**2**], is the free union of I and a one point space. No power X^λ is homogeneous because not all connected components of X^λ have the same cardinality. One must work a little harder to find a compact zero-dimensional space that is not powerhomogeneous. One reason is the interesting result of Motorov (cf. Arhangel'skii [**1**]), that every first countable compact zero-dimensional space X has X^ω homogeneous. Let $w(X)$ and $\pi w(X)$ stand for the weight and π-weight of the space X, respectively. The reader is encouraged to read a fundamental paper on nonhomogeneity by van Douwen [**2**] where he proves: If Y is an

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