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GENERALIZED FREE PRODUCTS OF π_c GROUPS

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ABSTRACT. A group G is said to be π_c if and only if for every pair of elements g_1 and g_2 of G either $g_1 = g_2^k$ for some integer k or there exists a normal subgroup N of finite index in G such that $g_1 \not\equiv g_2^z \bmod N$ for all integers z. In this note we prove that a certain generalized free product amalgamating a cyclic subgroup is π_c and then apply the result to show some one-relator groups are π_c.

1. Introduction. A group G is termed π_c if and only if for every pair of elements g_1 and g_2 of G either $g_1 = g_2^k$ for some integer k or there exists a normal subgroup N of finite index in G such that $g_1 \not\equiv g_2^z \bmod N$ for all integers z. Clearly, a π_c group is residually finite. However, the one-relator group $G = \langle a, b; a^{-1}ba = b^2\rangle$ is residually finite but not π_c [**1**]. Examples of π_c groups are the finite groups, free groups and finitely generated torsion-free nilpotent groups [**4, 5**].

In [**7**], Stebe proved that the generalized free products of isomorphic π_c groups amalgamating a cyclic subgroup are again π_c. In this note, we shall prove the following theorem.

Theorem. *Let A and B be π_c groups and let $a \in A$ and $b \in B$ where a and b have the same order. If A is $\langle a\rangle$-Pot and B is $\langle b\rangle$-Pot, then the generalized free product G of A and B amalgamating the subgroups $\langle a\rangle$ and $\langle b\rangle$ with $a = b$, is π_c.*

Modifying [**2**], a group G is termed $\langle x\rangle$-potent (or $\langle x\rangle$-Pot for short) for a nontrivial element x of G if and only if for every positive integer n (dividing the order of x if the order is finite) there exists a normal subgroup N of finite index in G such that xN has order exactly n in G/N. G is termed potent if it is $\langle x\rangle$-Pot for every nontrivial element x

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