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UNIFORM DENTABILITY, UNIFORM SMOOTHABILITY AND APPROXIMATIONS TO CONVEX SETS

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ABSTRACT. Uniform versions of dentability and smoothability are introduced, and shown to be dually related. It is shown that convex sets in spaces satisfying these properties can be approximated by the convex hull of a set of *uniformly sharp* corners in the former case and by the intersection of *uniformly flat* cones, generated by a point and the set, in the latter case.

Introduction. The notion of dentability in Banach spaces was introduced by Rieffel [**8**] and has been studied extensively since that time. One of the central results is that of Lindenstrauss (cf. [**7**]) which shows that if every bounded closed convex set in a Banach space X is dentable (the Radon-Nikodym property, abbreviated RNP), then every bounded closed convex set in X is the closed convex hull of its extreme points (the Krein-Milman property). This result was extended by Phelps [**7**] who showed that "extreme point" could be replaced by *strongly exposed point*. Loosely speaking, these results say that if X has the RNP, then a closed bounded convex set is the closed convex hull of its *corners*.

In [**5**], Finet defines a modulus of strong extremality for points of the unit ball which "measures how much a point is a strong extreme point of the unit ball." (See [**9**] or [**5**] for the definition of a strong extreme point.) It is then shown that the unit ball of a super-reflexive space can be approximated (arbitrarily close, using the Hausdorff metric) as the closed convex hull of a subset of the set of strong extreme points of the unit ball. The modulus of strong extremality for the points of this subset are uniformly bounded from below. (The bound depending on the closeness of the approximation.) Thus, thinking of the strong extreme points as corners, this says that the unit ball of a super-reflexive space can be approximated as the closed convex hull of a set of corners that are uniformly "sharp."

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