

K-THEORY OF ANALYTIC CROSSED PRODUCTS

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ABSTRACT. We prove the following theorem which is simultaneously a non-self-adjoint analogue of Conne's Thom Isomorphism and a generalization of a result of J. Peters. Suppose that G is a locally compact, compactly generated, abelian group and that Σ is a subsemigroup of G satisfying $(\Sigma^0)^- = \Sigma$ and $\Sigma \cap (-\Sigma) = \{0\}$. Then, for an arbitrary C^* -dynamical system (A, G, α) ,

$$K_i(\Sigma \times_\alpha A) \cong \begin{cases} K_i(A) & \text{if } G \text{ is discrete} \\ \{0\} & \text{otherwise} \end{cases} \\ (i = 0, 1).$$

1. Introduction. K -Theory has revolutionized the study of operator algebras in the last few years [2, 4, 1]. Most work is, however, devoted to C^* -algebras and relatively little is known on the K -theory for non-self-adjoint Banach algebras. A few results in this direction can be found in [12, 11].

We will concentrate our attention on the computation of the K -groups of analytic crossed products. We will use terminology, notation and basic facts on K -theory and crossed product used in [1, 10, 8]. We recall here some details about analytic crossed products for the sake of the reader's convenience.

Let A be a C^* -algebra, let G be a locally compact group with left Haar measure μ and let α be a continuous homomorphism from G into $\text{Aut}(A)$, the group of C^* -automorphisms of A with the topology of pointwise norm-convergence. Following the notation in [10], we denote the enveloping C^* -algebra of $L^1(G, A)$ by $G \times_\alpha A$ and call it the C^* -crossed product determined by the C^* -dynamical system (A, G, α) .

Let Σ be a closed subsemigroup of G containing the identity e of G satisfying the following conditions:

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