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CRITERIA FOR RIGHT DISFOCALITY OF AN nTH ORDER LINEAR DIFFERENCE EQUATION

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1. Introduction. We consider the nth order linear difference equation

$$Pu(m) \equiv \sum_{i=0}^{n} \alpha_i(m)u(m+i) = 0, \qquad m \in I \tag{1.1}$$

where I is an integer interval $I = [a, b] = \{a, a+1, a+2, \ldots, b\}$. We use the notation $I^k = [a, b+k]$ where k is an integer such that $k \geq a - b$, as used by Hartman [**5**]. We assume the coefficients $\alpha_i(m)$ are defined on I, with $\alpha_n(m) \equiv 1$ and that

$$(-1)^n \alpha_0(m) > 0 \qquad \text{for } m \in I. \tag{1.2}$$

In [**9**], necessary and sufficient conditions were given for right $(l, n-l)$-disconjugacy and left $(l, n-l)$-disconjugacy in terms of the coefficients $\alpha_i(m)$, $0 \leq i \leq n$, of equation (1.1), which lead to an improvement of a result of Hartman [**5**] for disconjugacy. Here we will give similar results for right ρ_l-disfocality. For the general nth order linear difference equation (1.1), we will give some necessary conditions in terms of the coefficients $\alpha_i(m)$ for right ρ_l-disfocality. In the special case $l = n - 1$, we will give necessary and sufficient conditions in terms of the coefficients $\alpha_i(m)$ for right ρ_{n-1}-disfocality. For the second order linear difference equation these results lead to necessary and sufficient conditions in terms of the coefficients $\alpha_i(m)$ for right disfocality.

The concept of disfocality for linear differential operators was introduced by Nehari [**7**]. Nehari showed that a certain generalized linear differential equation is disfocal if and only if the principal minors of a Wronskian matrix are positive.

More recently, Eloe [**1**] brought over to linear difference equations many of the results given by Muldowney [**6**] and Eloe and Henderson [**3**]

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