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ON THE EXTENSION OF DERIVATIONS TO SOME CLOSURES

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Introduction. Let A be a noetherian ring with integral closure $\overline{A}$ in its total quotient ring $k(A)$. When A is an integral domain, it is well known that any differentiation $\underline{D} = (1, D_1, \dots, D_i, \dots)$ of A extends to $\overline{A}$ [**3**, Section 2], but, generally, $\underline{D}$ doesn't extend to a ring lying between A and $\overline{A}$ (see [**4** ex. 1.1 or ex. 2.6]). Now let $A \subset B$ be noetherian integral domains. In Section 1, we consider some *closures* of A in B with respect to a given property. We prove that a differentiation $\underline{D}$ of A which extends to B also extends to the *u-closure* and to the *F-closure* of A in B; moreover, $\underline{D}$ extends to the *t-closure* of A in B whenever B is finite as an A-module. As regards (n-root)-*closure*, we show $\underline{D}$ can be extended under particular assumptions but not as a general rule (Section 1, Remark 1.9); we note that the (2,3)-*closure* has been already studied in [**5**].

The above problem can be considered from another point of view. If A is any noetherian ring, each ring between A and $\overline{A}$ can be seen as a suitable *closure* of A in $k(A)$ called Δ-*closure* and denoted with A^Δ (where Δ is a set of ideals of A, according to [**7**]). Since generally neither a differentiation of A nor an integrable derivation extends to A^Δ, we wonder when a derivation of A can be extended to A^Δ (Section 2). For any A^Δ, we give a sufficient (but not necessary) condition in order that a derivation D of A can be extended to A^Δ (Proposition 2.5), whereas, under suitable assumptions, we show that the extension of D to A^Δ can be characterized by certain properties of the conductor β of A in A^Δ. In particular, we prove the following. If A has (S_1)-property, D extends to $\overline{A}$ if and only if β is D-differential (Corollary 2.8).

Finally, in Section 3 we consider some classes of Δ-closures A^Δ and of derivations D of A satisfying the sufficient condition of Proposition 2.5, so that $D(A^\Delta) \subset A^\Delta$.

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