

## WALLMAN COMPACTIFICATION IN FTS

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**1. Introduction.** In this paper we construct a good extension of Wallman's compactification for all fuzzy topological spaces, and we prove that besides being a good extension of Wallman's original construction in TOP, our construction also extends the earlier construction in FNS which was given in [1]. FNS stands for the bireflective and coreflective subcategory of FTS consisting of all fuzzy neighborhood spaces.

Remarkably, in order to perform our construction we need to apply (in FTS) Shanin's generalization of Wallman's construction as given in [10]. The main advantages of our compactification over previous ones in, e.g., [2, 3, 8], are that, in the first place, it does not reduce to only a small class of fuzzy spaces (sometimes only to topologically generated spaces) and, in the second place, that it is given by a very explicit construction describing the closed fuzzy sets and a fortiori the convergence of prefilters in the compactification.

**2. Preliminaries.** We recall some concepts and notations which are required in this paper.  $I$  stands for the unit interval,  $I_0 := I \setminus \{0\}$  and  $I_1 := I \setminus \{1\}$ . We restrict our attention to classical fuzzy sets (i.e., functions with domain some set  $X$  and codomain  $I$ ) and classical fuzzy topological spaces in the sense of, e.g., [4]. Given a set  $X$ , a fuzzy topology on  $X$  is a collection  $\Delta \subset I^X$  which is closed under the taking of finite infima, arbitrary suprema and which contains all constants. A map  $f : (X, \Delta) \rightarrow (X', \Delta')$  between fuzzy topological spaces is said to be continuous if, for all  $\mu' \in \Delta'$ , we have  $\mu' \circ f \in \Delta$ . Fuzzy topological spaces and continuous maps form a topological category denoted FTS. This means, among other things, that initial and final structures exist in FTS. TOP is embedded in FTS as a full isomorphism closed subcategory which is at the same time bireflective and coreflective, i.e., the embedding has both a left and right adjoint. Given a fuzzy topological space  $(X, \Delta)$  its coreflection in TOP is determined by the topology  $\iota(\Delta) := \langle \{\mu^{-1}([\alpha, 1]) \mid \mu \in \Delta, \alpha \in I\} \rangle$ . If  $\Gamma$  is a collection of

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