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QUASILINEAR ELLIPTICITY AND JUMPING NONLINEARITIES

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ABSTRACT. Let $\Omega \subset \mathbf{R}^N$ be a bounded domain with smooth boundary. Also let

$$Qu = -D_i(a^{ij}(x,u,Du)D_j u) + b(x,u,Du)u.$$

Under five assumptions on the coefficients of Q (Caratheodory, symmetry, growth, ellipticity, and monotonicity) existence and nonexistence results for weak solutions to the generalized Dirichlet problem

$$Qu(x) = f(x,u) + t\Phi(x) + h(x) \quad \text{for } x \in \Omega;$$
$$u(x) = 0 \quad \text{for } x \in \partial\Omega$$

are established subject to jumping nonlinearity assumptions on $f(x,u)$ where $t \in \mathbf{R}$, $h \in L^\infty(\Omega)$, and $\Phi \in L^\infty(\Omega)$ is positive a.e. on Ω.

1. Introduction and statement of result. This paper will demonstrate some existence and nonexistence results for a quasilinear Dirichlet problem under Ambrosetti-Prodi, Berger-Podolak, Kazdan-Warner type assumptions (see [**1, 2** and **6** Theorems 3.4–3.8]). Let Ω be a bounded domain in $\mathbf{R}^N$ with smooth boundary denoted by $\partial\Omega$. Unless otherwise noted, all function spaces such as $L^2, W^{1,2}$, and H_0^1 will have domain Ω. We define the quasilinear operator

$$Qu = -D_i(a^{ij}(\cdot,u,Du)D_j u) + b(\cdot,u,Du)u$$

and study the existence and nonexistence of weak solutions to the generalized Dirichlet problem

$$(1.1_t) \qquad \begin{aligned} (Qu)(x) &= f(x,u(x)) + t\Phi(x) + h(x), \qquad x \in \Omega \\ u(x) &= 0, \qquad x \in \partial\Omega. \end{aligned}$$

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