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REMARKS ON SOME THEOREMS OF BANACH, McSHANE, AND PETTIS

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ABSTRACT. In [**12**] B.J. Pettis gives several results which sharpen results of McShane [**7**]. These results use properties related to the ideal of meager or first category subsets. An *ideal* is a collection of subsets closed under the operations of subset and finite union. In this paper we identify the crucial properties of the ideal of meager sets which make these results possible and extend the results to other ideals which possess these properties. A well-known result of Banach [**1**], concerning subgroups of a topological group is extended and two theorems concerning continuity of homomorphisms and linear transformations are given as applications.

1. Introduction. In [**12**] B.J. Pettis gives several results which sharpen results of E.J. McShane [**7**] which in turn extend a well-known theorem of Banach [**1**]. These results use properties related to the ideal of meager or first category subsets. In this paper we identify the crucial properties of this ideal which make these results possible and extend the results to other ideals which possess these properties.

An *ideal* ([**6, 15, 16**]) $\mathcal{I}$ on a topological space (X,τ) is a collection of subsets of X which satisfies the following two properties: (1) $A\in\mathcal{I}$ and $B\subseteq A$ implies $B\in\mathcal{I}$ (heredity), and (2) $A\in\mathcal{I}$ and $B\in\mathcal{I}$ implies $A\cup B\in\mathcal{I}$ (finite additivity). If, in addition, $\mathcal{I}$ satisfies (3) $\{A_n : n=1,2,3,\dots\}\subseteq\mathcal{I}$ implies $\cup\{A_n : n=1,2,3,\dots\}\in\mathcal{I}$ (countable additivity), then $\mathcal{I}$ is called a *σ-ideal.*

Let $\mathcal{P}(X)$ denote the power set of X. Given a topological space (X,τ) and an ideal $\mathcal{I}$ on X, a set operator $(\quad)^*(\mathcal{I},\tau)$: $\mathcal{P}(X)\to\mathcal{P}(X)$, called the *local function of $\mathcal{I}$ with respect to τ* in [**16**] is defined as follows: for $A\subseteq X$, $(A)^*(\mathcal{I},\tau)=\{x\in X : U\cap A\notin\mathcal{I}$ for every $U\in\tau(x)\}$, where $\tau(x)=\{U\in\tau : x\in U\}$. A Kuratowski closure

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