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POSITIVE SOLUTIONS AND J-FOCAL POINTS FOR TWO POINT BOUNDARY VALUE PROBLEMS

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ABSTRACT. Cone theory is applied to a class of two point boundary value problems for ordinary differential equations. Criteria for the existence of extremal points are obtained. These criteria are in terms of the existence of nontrivial solutions that lie in a cone, and in terms of the spectral radius of an associated compact linear operator.

1. Introduction. Let $n > 1$ be a positive integer, and let $\alpha < \beta$, $k \in \{1, \dots, n-1\}$, and $j \in \{0, \dots, k\}$ be given. Let $p_i \in C[\alpha, \beta]$, for $i = 0, \dots, j$, and consider the linear ordinary differential equation,

$$y^{(n)} = \sum_{i=0}^{j} p_i(x) y^{(i)}, \qquad \alpha \le x \le \beta. \tag{1.1}$$

We shall be concerned with extremal point properties of (1.1) with respect to the family of two point boundary conditions,

$$\begin{aligned} y^{(i)}(\alpha) &= 0, \qquad i = 0, \dots, k-1, \\ y^{(i)}(b) &= 0, \qquad i = j, \dots, n-k+j-1, \end{aligned} \tag{1.2$_b$}$$

where $b \in (\alpha, \beta]$. Note that, if $j = 0$, then (1.2$_b$) represents conjugate boundary conditions, if $j = k$, then (1.2$_b$) represents right focal boundary conditions, and if $j \in \{1, \dots, k-1\}$, then (1.2$_b$) represents boundary conditions that are "between" conjugate and right focal boundary conditions.

Definition. $b_0 \in (\alpha, \beta]$ is the *j-focal point of* (1.1) *corresponding to* (1.2$_b$) if, and only if,

$$b_0 = \inf\{b > \alpha \mid (1.1), (1.2_b) \text{ has a nontrivial solution}\}.$$

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