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# A THEOREM ON REPRODUCING KERNEL HILBERT SPACES OF PAIRS

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ABSTRACT. In this paper we study reproducing kernel Hilbert and Banach spaces of pairs. These are a generalization of reproducing kernel Hilbert spaces and, roughly speaking, consist of pairs of Hilbert (or Banach) spaces of functions in duality with respect to a sesquilinear form and admitting a left and a right reproducing kernel. We first investigate some properties of these spaces of pairs. It is then proved that to every function $K(z,w)$ analytic in $z$ and $w^*$ there is a neighborhood of the origin that can be associated with a reproducing kernel Hilbert space of pairs with left reproducing kernel $K(z,w)$ and right reproducing kernel $K(w,z)^*$.

**1. Introduction.** Hilbert spaces of functions with bounded point evaluations (reproducing kernel Hilbert spaces, the definition is recalled in the sequel) play an important role in a number of areas in analysis (see, e.g., [**7, 8, 10, 21**]). The special case of reproducing kernels of the form

$$K(z,w) = \frac{X(z)JX(w)^*}{\rho_w(z)}, \tag{1}$$

where

(a) $J$ is a $\mathbf{C}^{m\times m}$ matrix subject to $J = J^* = J^{-1}$,

(b) $X$ is a $\mathbf{C}^{k\times m}$ valued function meromorphic in $\Delta_+$, where $\Delta_+$ denotes either the open unit disk $\mathbf{D}$ or the open upper half plane $\mathbf{C}^+$,

and

(c) the function $\rho$ is defined by

$$\rho_w(z) = \begin{cases} 1 - zw^* & \text{if } \Delta_+ = \mathbf{D} \\ -2\pi i(z - w^*) & \text{if } \Delta_+ = \mathbf{C}^+, \end{cases}$$

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