

ALGEBRAIC CHARACTERIZATION OF DISTRIBUTIONS OF RAPID GROWTH

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ABSTRACT. In this paper we obtain algebraic characterization of the space \mathcal{K}'_M of distributions which grow no faster than $\exp(M(kx))$, and the space $O'_c(\mathcal{K}'_M : \mathcal{K}'_M)$ of its convolution operators; where M is an index function and k is a positive integer. We show that $\mathcal{K}'_M, O'_c(\mathcal{K}'_M : \mathcal{K}'_M)$ are homeomorphic to the vector spaces of module homeomorphisms $\text{Hom}_{\mathcal{K}_M}(\mathcal{K}_M, O_c)$ and $\text{Hom}_{\mathcal{K}_M}(\mathcal{K}_M, \mathcal{K}_M)$, respectively. The relation between these results and invertibility of convolution operators in \mathcal{K}'_M is being discussed.

1. Introduction. Algebraic characterization of the space \mathcal{D}' of Schwartz distributions was obtained by Struble [7], he proved that \mathcal{D}' is homeomorphic to the vector space of homeomorphisms from \mathcal{D} into \mathcal{E} over \mathcal{D} , when both spaces are provided with the topology of pointwise convergence, where \mathcal{D} and \mathcal{E} are the Schwartz spaces of test functions. Abdullah [1] obtained algebraic characterizations of the space \mathcal{K}'_p of distributions which grow no faster than $\exp(k|x|^p)$; $p \geq 1, k \geq 0$ and the space $O'_c(\mathcal{K}'_p : \mathcal{K}'_p)$ of its convolution operators. It has been shown that \mathcal{K}'_p is homeomorphic to the vector space $\text{Hom}_{\mathcal{K}_p}(\mathcal{K}_p, O_c(\mathcal{K}'_p : \mathcal{K}'_p))$ of homeomorphism from \mathcal{K}_p into $O_c(\mathcal{K}'_p : \mathcal{K}'_p)$ over \mathcal{K}_p and that $O'_c(\mathcal{K}'_p : \mathcal{K}'_p)$ is homeomorphic to the ring $\text{Hom}_{\mathcal{K}_p}(\mathcal{K}_p, \mathcal{K}_p)$ of homeomorphisms from \mathcal{K}_p into itself over \mathcal{K}_p . All the spaces involved were provided with their strong topologies. In this paper we extend the results of [1] to the spaces \mathcal{K}'_M and $O'_c(\mathcal{K}'_M : \mathcal{K}'_M)$, where \mathcal{K}'_M is the space of distributions which grow no faster than $\exp(M(kx))$ and $O'_c(\mathcal{K}'_M : \mathcal{K}'_M)$ is the space of its convolution operators (see the next section for definitions). We show that \mathcal{K}'_M is homeomorphic to the vector space $\text{Hom}_{\mathcal{K}_M}(\mathcal{K}_M, O_c(\mathcal{K}'_M : \mathcal{K}'_M))$ of module homeomorphisms from \mathcal{K}_M into $O_c(\mathcal{K}'_M : \mathcal{K}'_M)$ over \mathcal{K}_M , and that $O'_c(\mathcal{K}'_M : \mathcal{K}'_M)$ is homeomorphic to the ring $\text{Hom}_{\mathcal{K}_M}(\mathcal{K}_M, \mathcal{K}_M)$ of module homeomorphisms from \mathcal{K}_M into itself over \mathcal{K}_M . On the one hand, the topological spaces \mathcal{K}'_M and $O'_c(\mathcal{K}'_M : \mathcal{K}'_M)$ are assigned algebraic structures. And, on the other

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