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SUBGROUP SEPARABILITY OF CERTAIN HNN EXTENSIONS

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ABSTRACT. We show that certain HNN extensions are subgroup separable and then apply the result to get a characterization for the Baumslag-Solitar groups to be subgroup separable and some other results.

1. The residual finiteness and hopficity of the one-relator groups $G_{k,l} = \langle t, a; t^{-1}a^k t = a^l \rangle$, now called the Baumslag-Solitar groups, were exhaustively studied and completely characterized by Baumslag and Solitar [**2**], Meskin [**7**] and Collins and Levin [**3**]. Their results can be summarized as follows:

Theorem 1. *Let $G_{k,l} = \langle t, a; t^{-1}a^k t = a^l \rangle$. Then $G_{k,l}$ is residually finite if and only if $|k| = 1$ or $|l| = 1$ or $|k| = |l|$ and $G_{k,l}$ is hopfian if and only if $|k| = 1$ or $|l| = 1$ or $\pi(k) = \pi(l)$, where $\pi(n)$, for a nonzero integer n, denotes the set of prime divisors of n.*

In the note we shall characterize the groups $G_{k,l}$ with regards to subgroup separability. We shall prove the following:

Theorem 2. *Let $G_{k,l} = \langle t, a; t^{-1}a^k t = a^l \rangle$. Then $G_{k,l}$ is subgroup separable if and only if $|k| = |l|$.*

Theorem 2 will follow from Theorems 1, 3 and 4. Theorem 3, which is our main result, partially extends Theorem 1 of Andreadakis, Raptis and Varsos [**1**].

The notations used here are standard. In addition, the following will be used. Let G be a group.

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