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WEAK AND NORM CONVERGENCE ON THE UNIT SPHERE

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ABSTRACT. In this paper we prove that the properties (KK) and (K) in a Banach space are stable for the generalized Banach products. We also establish some relationship between these and other properties related with weak and norm convergence on the unit sphere of a Banach space.

1. Notations. We follow standard terminology that can be found in [**1**]. Let $(X, ||\cdot||)$ be a Banach space. B_X denotes its closed unit ball, S_X the unit sphere, X^* the topological dual of X. If (x_n) is a sequence in X, let $\text{sep}\,(x_n) = \inf\{||x_n - x_m||, \text{ for all } n, m \in \mathbf{N},\ m \neq n\}$. We denote by $\mathcal{F}_f(I)$ the family of the finite subsets of set I. $\mathbf{K}$ denotes the field of real or complex numbers.

2. Introduction. Several classes of Banach spaces have been introduced in the past according to the fulfillment of certain properties related with weak and norm convergence on the unit sphere of a Banach space $(X, ||\cdot||)$. We can mention:

(KK): *Kadec-Klee Property*: If (x_n) is a sequence of elements in X converging weakly to an element x in X such that $||x_n|| \to ||x||$, then (x_n) converges to x in norm, (i.e., for sequences on the unit sphere weak and norm convergence coincide).

(K): *Kadec Property*: The weak and the norm topology coincide on the unit sphere.

(α): *Property* (α) (Rolewicz [**6**]): Given an element $f \in X^*$ such that $||f|| = 1$ and $\varepsilon > 0$, let

$$S(f, \varepsilon) = \{x : x \in B_X, f(x) \geq 1 - \varepsilon\}.$$

The Kuratowski index of noncompactness $\alpha(A)$ of a subset A of X is defined as the infimum of all positive numbers r such that A can be

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