ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 23, Number 1, Winter 1993

STURMIAN THEORY FOR NONSELFADJOINT SYSTEMS

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ABSTRACT. The theory of μ_0-positive operators is used to systematically develop the Sturmian properties of the second order system (1) $(r(t)x')' + q(t)x = 0$, where $r(t)$ and $q(t)$ are $n \times n$ matrices of continuous functions on $[a,b]$. Since no symmetry assumptions are made on either of the matrices $r(t)$ or $q(t)$, (1) will in general be nonselfadjoint. However, all results are new even if (1) is selfadjoint. It is assumed that $r^{-1}(t)$ and $q(t)$ are *positive* with respect to some cone, K, in Euclidean space with nonempty interior K^0. With some additional assumptions on $r(t)$, the following basic result is given. If b is the first conjugate point to a, then there exists a unique (up to multiplication by a constant) nontrivial solution, $x(t)$, to (1) with $x(a) = 0 = x(b)$ and $x(t) \in K^0$ on (a,b).

1. Introduction. In this paper the theory of μ_0-positive operators defined on a Banach space equipped with a cone is used to develop certain Sturmian properties of the system of second order differential equations

$$(r(t)x')' + q(t)x = 0, \tag{1}$$

where $r(t)$ and $q(t)$ are $n \times n$ matrices of continuous functions on $[a,b]$, $a \geq 0$, and $r(t)$ is nonsingular for all $t \in [a,b]$ and $\int_a^t r^{-1}(s)\,ds$ is nonsingular for all $t \in (a,b]$. Since no symmetry assumptions are made on either of the matrices $r(t)$ or $q(t)$, (1) will in general be nonselfadjoint. However, all results presented here are new even if (1) is selfadjoint.

Equation (1) with $r(t) \equiv E$, the identity matrix, has been studied recently by a number of people (see [**1**–**12**, **14**, **16**–**21**]). It needs to be emphasized, however, that nobody has obtained results for conjugate points for the more general equation (1). Keener and Travis [**10**] used μ_0-positive operators to study conjugate and focal points of (1) when

Received by the editors on January 25, 1990, and in revised form on September 18, 1990.