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RELATING DIFFERENT CONDITIONS FOR THE POSITIVITY OF THE SCHRÖDINGER OPERATOR

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ABSTRACT. The following article directs proofs that sufficient conditions for the positivity of the Schrödinger operator due to C. Fefferman and Chang, Wilson, and Wolff imply a necessary and sufficient condition of Kerman-Sawyer. The method is by reduction to dyadic case, Calderon-Zygmund decomposition, and, in one case, the use of Orlicz norms.

This article presents some direct proofs between several different conditions which imply the positivity of the Schrödinger operator, $-\Delta - (1/c)v$, where $v \geq 0$. If $L = -\Delta - (1/c)v$ is essentially self-adjoint, then L being a positive operator is equivalent to the following inequality:

$$(*) \qquad \int_{\mathbf{R}^d} u^2(x)v(x)\,dx \leq c\int_{\mathbf{R}^d} |\nabla u(x)|^2\,dx \qquad \forall\, u \in C_0^\infty,$$

as can be seen by an integration by parts:

$$\begin{aligned}\langle(-\Delta - \frac{1}{c}v)u, u\rangle &= \int(-\Delta u)u - \frac{1}{c}\int u^2 v\\ &= \int |\nabla u|^2 - \frac{1}{c}\int u^2 v\\ &\geq 0 \Leftrightarrow (*) \text{ holds.}\end{aligned}$$

In his paper, "The Uncertainty Principle," [**5**], C. Fefferman raises the question: What conditions on v imply $(*)$? In [**5**] the following condition (a) is shown to be sufficient for $(*)$:

(a) There exists $p > 1$ for all cubes Q

$$\left(\frac{1}{|Q|}\int_Q v^p\right)^{1/p} \leq \frac{c}{l^2(Q)}$$

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