ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 23, Number 1, Winter 1993

NOTE ON A NONLINEAR EIGENVALUE PROBLEM

PETER LINDQVIST

ABSTRACT. This note complements some known facts about the ordinary differential equation $(|u'|^{p-2}u')'+\lambda|u|^{p-2}u = 0$. The eigenvalues exhibit a fascinating dependence on the exponent p, namely, $\sqrt[p]{\lambda_p} = \sqrt[q]{\lambda_q}$ for conjugate exponents. In terms of the Rayleigh quotients,

$$\min_u \frac{||u'||_p}{||u||_p} = \min_v \frac{||v'||_q}{||v||_q}, \qquad \frac{1}{p}+\frac{1}{q}=1.$$

Corresponding eigenfunctions are related for conjugate exponents. We shall express this dependence in a nice formula.

1. Introduction. The minimum λ_p of the Rayleigh quotient

$$\frac{\int_a^b |u'(x)|^p\,dx}{\int_a^b |u(x)|^p\,dx}, \qquad 1 < p < \infty \tag{1}$$

taken among all real-valued functions $u \in C^1[a,b]$ with $u(a) = u(b) = 0$ is equal to the first eigenvalue λ of the equation

$$\frac{d}{dx}(|u'|^{p-2}u') + \lambda|u|^{p-2}u = 0. \tag{2}$$

(The resulting sharp estimate $\sqrt[p]{\lambda_p}||u||_p \le ||u'||_p$ is called Wirtinger's inequality in the classical case $p = 2$, when the equation reduces to $u'' + \lambda u = 0$.) The existence of eigenvalues and eigenfunctions has been considered in [**1**, Theorem 4.4]. This problem has been thoroughly studied by M. Ôtani. He has explicitly determined all eigenvalues and described the eigenfunctions and their zeros, cf. [**4**]. These results are so exhaustive that it seems difficult to add anything

Mathematics subject classifications. 34A34, 34B99.
Received by the editors on October 20, 1990, and in revised form on March 8, 1991.