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ON JACOBIAN n-TUPLES IN CHARACTERISTIC p

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0. Introduction. Let k be a field and $A = k[x_1, \dots, x_n]$. For $(F_1, \dots, F_n) \in A^n$, let $j(F_1, \dots, F_n)$ denote the determinant of the $n \times n$ Jacobian matrix of $F_1, \dots, F_n$ with respect to the x_i, $1 \le i \le n$. We say that $(F_1, \dots, F_n) \in A^n$ is a *Jacobian n-tuple* if $j(F_1, \dots, F_n) \in k^*$, the multiplicative group of nonzero elements in k. The Jacobian conjecture states:

(0.1) If $\operatorname{char}(k) = 0$, then $j(F_1, \dots, F_n) \in k^*$ implies $k[F_1, \dots, F_n] = A$.

This conjecture, introduced by O.H. Keller [**5**] in 1939, has remained unsolved, for $n \ge 2$, and (0.1) is not true if the characteristic of k is positive ([**1**, p. 118]). Nonetheless, we feel that the study of Jacobian n-tuples when the characteristic is positive may contribute to a better understanding of Jacobian n-tuples in characteristic 0 for two reasons. Firstly, E. Connell and L. van den Dries have shown that the general Jacobian conjecture is equivalent to proving (0.1) for the case where $F_1, \dots, F_n$ are cubic polynomials with integer coefficients (see (1.1) below); thus, information we obtain in characteristic p on cubic Jacobian n-tuples may be related backwards to the characteristic 0 situation. Secondly, S. Abhyankar proved various equivalent formulations of (0.1) in the $n = 2$ case in terms of Newton Polygons, points at infinity, and the degrees of F_1 and F_2 in [**1**] (see (1.3) below). Since for each Jacobian pair in characteristic 0, there are corresponding Jacobian pairs with matching supports in characteristic p for almost all p, our hope is to eventually shed some light on the $n = 2$ case of (0.1) (see (1.2) below).

In this paper we give some new characterizations of Jacobian n-tuples in characteristic p in terms of the differential operator $\nabla =$

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