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ALMOST REGULAR INTEGER FIBONACCI PENTAGONS

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Dedicated to the memory of Vern E. Hoggatt, Jr.

Introduction. An *integer pentagon* is a pentagon with integer sides and diagonals. Small examples of *integer pentagons* are scarce but two are shown in Figure 1. The one designated as A appears in Muller [**2**] and in Harborth and Kemnitz [**1**] and is claimed in the latter article to be the *integer pentagon* of smallest diameter (another at least as small exists). The one designated as B may never before have appeared in print. Both of these have the pleasing property of having four distances the same, three other distances the same, and two of the remaining three distances the same as well as having symmetry with respect to the bisector of the base.

Let F_n represent the n-th Fibonacci number and L_n represent the n-th Lucas number. Both of the pentagons of Figure 1 are generated by $F_3 = 2$ in the sense that $8 = 2^3$; $7 = 8 - F_1$; $6 = F_1F_3F_4$; $4 = F_2F_3F_3$; $3 = 8 - F_5$; $10 = F_2F_3F_5$ and $12 = F_3F_3F_4$.

It is the purpose of this paper to display for each F_n, $n \geq 3$, two integer pentagons generated by Fibonacci numbers (one, the type of A, and, the other, the type of B).

The sequences. To generalize, consider Figure 2. The values of $n \geq 3$ are:

$$\begin{aligned}
a_{n-2} &= F_{n-1}F_n^2\\
b_{n-2} &= F_{n-2}F_nF_{n+1}\\
c_{n-2} &= F_n^3\\
d_{n-2} &= F_n^3 + (-1)^n F_{n-2}\\
h_{n-2} &= F_{n+1}F_n^2\\
k_{n-2} &= F_{n-1}F_nF_{n+2}\\
g_{n-2} &= F_n^3 + (-1)^n F_{n+2}.
\end{aligned}$$

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