EQUIVALENCE THEOREMS

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1. Introduction. This is a revised and slightly expanded version of a lecture given at the workshop on abelian groups held at the University of Connecticut in October 1989. The lecture was actually a general departmental colloquium held during the workshop, and therefore the exposition is as nontechnical as possible. Because it is intended for a more general audience, this paper is more basic and perhaps a little less formal than most publications.

To begin, I would like to ask my audience and readers, in order to be properly motivated, to accept principally the premise set forth in the following quotation from Thomas Hungerford's *Algebra*. "Ideally the goal in studying groups is to classify all groups up to isomorphism, which in practice means finding necessary and sufficient conditions for two groups to be isomorphic."

If the above level of achievement is thought of as the pinnacle of the theory of groups, there certainly are many other high points of interest that surround and support the pinnacle. The whole mass, as it were from the foot of the mountain to the top, dealing directly with the question of when two groups are isomorphic or are related in some weaker sense is known as structure theory. This is the oldest and most settled side of the mountain (of abelian group theory), particularly for torsion groups. The other (some say softer) side of the mountain is the homological face, which was first explored seriously by D. Harrison [**11**] in 1959; it should be understood that we are going to be dealing exclusively with abelian groups. However, before we decree that all groups are abelian, let us mention in passing that although the strategies and techniques employed by abelian group theorists and nonabelian group theorists are quite different, the two camps share the common goal, if on different paths, of reaching the pinnacle identified above. Consider, for example, the effort in the 60's and 70's to classify finite groups.