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BOUNDED ANALYTIC FAMILIES OF CAUCHY-STIELTJES INTEGRALS

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1. Introduction. Let $\Delta = \{z \in \mathbf{C} : |z| < 1\}$ and $\Lambda = \{z : |z| = 1\}$. Let $\mathcal{M}$ denote the set of (finite) complex-valued Borel measures on Λ. For $\alpha > 0$, let $\mathcal{F}_\alpha$ denote the family of functions f having the property that there exists $\mu \in \mathcal{M}$ such that

$$f(z) = \int_\Lambda \frac{1}{(1-xz)^\alpha}\, d\mu(x) \tag{1}$$

for $|z| < 1$. If $f \in \mathcal{F}_\alpha$, let $||f||_{\mathcal{F}_\alpha} = \inf ||\mu||$ where μ varies over all members of $\mathcal{M}$ for which (1) holds and where $||\mu||$ denotes the total variation of μ. Then $\mathcal{F}_\alpha$ is a Banach space with respect to this norm and the usual addition of functions and multiplication by complex numbers.

Properties of $\mathcal{F}_\alpha$ were studied in [**8**] and [**5**], where the related family denoted $\mathcal{F}_0$ was introduced. A function $f \in \mathcal{F}_0$ provided that there exists $\mu \in \mathcal{M}$ such that

$$f(z) = f(0) + \int_\Lambda \log\left(\frac{1}{1-xz}\right) d\mu(x) \tag{2}$$

for $|z| < 1$. The family $\mathcal{F}_1$ has been studied extensively. The survey article [**1**] gives a number of references in this area.

A Banach space of analytic functions is defined in this paper for each real number α. It is shown that when $\alpha \geq 0$ the space is equivalent to $\mathcal{F}_\alpha$. This provides a natural extension of $\mathcal{F}_\alpha$ for $\alpha < 0$. The results obtained also clarify why $\mathcal{F}_0$ is an appropriate choice for $\mathcal{F}_\alpha$ when $\alpha = 0$.

Let α be a real number. Define the function G_α by

$$G_\alpha(z) = \sum_{n=1}^{\infty} n^{\alpha-1} z^n \tag{3}$$

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