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NILPOTENCE OF PRODUCTS OF NONNEGATIVE MATRICES

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ABSTRACT. Given m nonnegative n-by-n matrices $A_1, A_2, \ldots, A_m$, we consider the circumstances in which the product $A_1A_2 \ldots A_m$ is nilpotent and also the stronger condition that $A_1, A_2, \ldots, A_m$ are simultaneously permutation similar to strictly upper triangular matrices. These eventualities coincide in the case of a single matrix, but they differ for several matrices and are each characterized in a variety of ways.

1. Introduction and notations. We consider the question of which sequences of m nonnegative n-by-n matrices have a nilpotent product. This problem comes to our attention from the study of the structural properties of discrete time positive periodic linear systems. Given a discrete-time periodic linear system

$$x(k+1) = A(k)x(k) + B(k)u(k) \tag{1}$$

in which $A(k)$ and $B(k)$ are nonnegative N-periodic matrices of sizes n-by-n and n-by-p, respectively, we consider the n discrete time invariant linear systems associated with (1)

$$x_s(k+1) = A_s x_s(k) + B_s u_s(k), \qquad s = 0, 1, \ldots, N-1. \tag{2}$$

Recently, in [**1**], there appeared a characterization of positive controllability of (1) in terms of positive controllability of the systems (2); more precisely, it was shown that "The positive periodic system (1) is completely positive orthant controllable at s if and only if the positive invariant system (2) corresponding to index s is completely positive orthant controllable, $s = 0, 1, \ldots, N-1$." Further, it is known (see

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