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EXTENSIONS AND INTERNAL STRUCTURE

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ABSTRACT. This paper introduces a kind of grid which helps to organize the information about T_0-extensions of topological spaces. The kinds of extensions a space can support are closely controlled by its internal structure. For example, a T_2-space cannot have a *compact* T_2-extension unless it is also completely regular. But every extension induces a trace system of filters on the original space; moreover, these filter systems can be arranged into proximity classes. Thus, the proximities form a kind of x-axis, and each proximity class is a kind of y-axis.

Using this grid, we can "plot" the Stone-Čech, the Wallman, and the one-point compactification. They all turn out to have the same "height." In addition, we can identify new classes of compactifications. For example, each proximity class has a largest filter system, which consists of all the open filters which are in some sense compatible with the proximity. By taking only the filters which are minimal in some sense related to the proximity, we obtain a compactification which is highly separated. If the proximity is dense and separated, then this compactification is the unique T_2-compactification induced by the proximity. Of course, in such a case the original space must be T_2 and completely regular. This is just a sample of the kinds of results obtained. It is hoped that this idea of a grid will continue to shed light on the ways a space can be extended.

1. Filter systems and extensions. Let X be a T_0-space. An extension (e, Y) induces a system of open filters on X via the pullbacks of the neighborhood filters under e. This system is known as the *trace system* of (e, Y). It turns out that each system of open filters on X which includes all the neighborhood filters is the trace system for some T_0-extension. This leads to the following.

Definition 1.1. A *filter system* on a T_0-topological space X is a family of open filters which includes all the neighborhood filters.

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