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REDUCIBLE EPIDEMICS: CHOOSING YOUR SADDLE

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1. Introduction. The propagation of infection in spatial models for a deterministic n-type $S \to I \to R$ epidemic with a nonreducible infection matrix is now fairly well understood. Under certain conditions the equations admit wave solutions travelling with each speed c greater than or equal to a minimum speed c_0. The conditions are that the Perron-Frobenius root of the infection matrix is greater than one, and that the contact distributions are exponentially dominated in the forward tail. The solution at each speed $c > c_0$ has been proved to be unique modulo translation. For the critical case $c = c_0$, when c_0 is positive, the wave solution has been proved, except in an exceptional case, to be unique modulo translation.

A saddle-point approximation can be used to give an indication of the asymptotic speed of propagation. The result for the one-type simple epidemic was obtained by Daniels [**2**]. A rigorized approach to the saddle-point method, and the result for the speed of propagation for the n-type epidemic are given in Radcliffe and Rass [**6**]. This suggests that the asymptotic speed of propagation is in fact c_0, the minimum speed for which wave solutions exist. That this is the case has been proved by exact analytic methods. The one-type case appears in Aronson [**1**], Diekmann [**3**] and Thieme [**9**], and the n-type cases appears in Radcliffe and Rass [**7**]. Note that these results all assume radially symmetric contact distributions.

Recently the authors (Radcliffe and Rass [**8**]) have investigated the possible wave solutions for an $S \to I \to R$ model with a reducible infection matrix. Some interesting results were obtained. In particular, under certain conditions, more than one wave solution exists at a particular speed; these wave solutions affecting both types and having different behavior in their tails.

It is shown in Section 2 that the spatial models for the $S \to I \to R$ and $S \to I \to S$ epidemics both lead to the same equations for the spread of infection in the forward tail of the epidemic. Thus, the

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