ON A CLASSICAL BOUNDARY VALUE PROBLEM INVOLVING A SMALL PARAMETER

STEPHEN J. KIRSCHVINK

ABSTRACT. Differential inequality techniques are used to provide accurate information throughout the interval \([a, b]\) on boundary layer solutions of the problem

\[
\varepsilon y'' = f(t, y) y' + g(t, y) \quad \text{for} \quad a \leq t \leq b,
\]

\[
y(a) = A \quad \text{and} \quad y(b) = B,
\]

subject to weak regularity requirements on the data (\(\varepsilon > 0\) is a small positive parameter). Such accurate information has been previously obtained by asymptotic expansion techniques coupled with the contraction mapping method, but only subject to more severe regularity requirements on the data, whereas the differential inequality technique has previously given such accurate information subject to weak regularity requirements, but only outside the boundary layer, with a loss of accuracy occurring inside the boundary layer. The detailed approximations of solutions obtained here may be very useful in studying solutions with other types of singularity perturbed behavior, such as shock or interior layer behavior. Problems of this type arise in fluid dynamics.

1. Introduction. In this paper is studied the singularly perturbed scalar boundary value problem

\[
\varepsilon y'' = f(t, y) y' + g(t, y), \quad a < t < b,
\]

\[
y(a) = A \quad \text{and} \quad y(b) = B,
\]

where \(\varepsilon > 0\) is a small parameter, and where \(y, f, g, A,\) and \(B\) are real valued quantities. This problem has been much studied in the literature, mainly by the method of differential inequalities and the method of contraction mappings. The former method has provided the existence of solutions and detailed information on solutions away from the boundary layer, subject to relatively weak smoothness requirements.

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