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SOME EXAMPLES OF MIXING RANDOM FIELDS

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ABSTRACT. Several classes of strictly stationary random fields are constructed, with various combinations of "strong mixing" properties. The purpose is to "separate" various mixing assumptions that are used in the literature on limit theory for random fields.

1. Introduction. Suppose $(\Omega, \mathcal{F}, P)$ is a probability space. For any two σ-fields $\mathcal{A}$, $\mathcal{B} \subset \mathcal{F}$ define the following measures of dependence:

$$\alpha(\mathcal{A},\mathcal{B}) := \sup_{A\in\mathcal{A}, B\in\mathcal{B}} |P(A\cap B) - P(A)P(B)|,$$

$$\rho(\mathcal{A},\mathcal{B}) := \sup_{f\in L_2(\mathcal{A}), g\in L_2(\mathcal{B})} |\mathrm{Corr}\,(f,g)|,$$

$$\beta(\mathcal{A},\mathcal{B}) := \sup \frac{1}{2}\sum_{i=1}^{I}\sum_{j=1}^{J} |P(A_i\cap B_j) - P(A_i)P(B_j)|$$

where this last sup is taken over all pairs of partitions $\{A_1, \dots, A_I\}$ and $\{B_1, \dots, B_J\}$ of Ω such that $A_i \in \mathcal{A}$ for each i and $B_j \in \mathcal{B}$ for each j. The following inequalities are elementary:

$$4\alpha(\mathcal{A},\mathcal{B}) \le \rho(\mathcal{A},\mathcal{B}) \le 1, \quad \text{and} \qquad (1.1)$$
$$2\alpha(\mathcal{A},\mathcal{B}) \le \beta(\mathcal{A},\mathcal{B}) \le 1.$$

Suppose d is a positive integer. For each $l := (l_1, \dots, l_d) \in \mathbf{Z}^d$ denote the usual Euclidean norm $||l|| := (l_1^2 + \cdots + l_d^2)^{1/2}$. For any two nonempty disjoint subsets $S, T \subset \mathbf{Z}^d$, denote the distance between them by

$$\mathrm{dist}\,(S,T) := \inf_{s\in S, t\in T} ||s-t||.$$

Now suppose $X := (X_t, t\in\mathbf{Z}^d)$ is a strictly stationary random field on our probability space $(\Omega, \mathcal{F}, P)$. For each real number $r \ge 1$, and

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