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TWO WEIGHTED (L^p, L^q) ESTIMATES FOR THE FOURIER TRANSFORM

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0. Introduction and notation. In this paper we continue to study the two-weight problem for the Fourier transform. The problem is for given p and q with $1 < p \leq q < \infty$, to determine necessary and sufficient conditions on w and v so that

$$\left(\int_{\mathbf{R}^n} |\hat{f}(x)|^q w(x)\, dx \right)^{p/q} \leq C \int_{\mathbf{R}^n} |f|^p v(x)\, dx, \tag{0.1}$$

where C is a positive constant independent of f.

In the case where w and $1/v$ are radial and symmetrically decreasing, this was completely solved in Theorem 2 of [**4**]. There the Fourier transform problem was reduced to the two-weight problem for the Hardy operator.

In two dimensions ($n = 2$) for $p = q = 2$, Kerman and Sawyer solved the problem when w and $1/v$ are symmetrically decreasing in each of their variables. Here they showed that the Fourier transform problem can be reduced to a two-weight problem for the two-dimensional Hardy operator, solved by Sawyer in [**6**], where he also presented these results.

Heinig and Sinnamon in [**2**] were able to generalize these results of Kerman and Sawyer to n-dimensions for conjugate exponents, where w decreases in each of its variables and v has the special form $v(x) = w(1/x)^{p/q}$ (note $1/x = (1/x_1, 1/x_2, \dots, 1/x_n)$). Furthermore, the necessary and sufficient conditions they obtain are quite easy to apply.

We obtain the following results. In Section 2, in n-dimension, for weights w and $1/v$ that are symmetrically decreasing in each of their variables, we reduce the Fourier transform problem to a two-weight problem for the n-dimensional Hardy operator. Here $p \leq q$, with q a positive even integer. We should point out, though, that the Hardy problem is still open in three or higher dimensions.

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