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BLOCH TYPE SPACES OF ANALYTIC FUNCTIONS

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1. Introduction. Let $\mathbf{D}$ be the open unit disk in the complex plane $\mathbf{C}$. The Bloch space of $\mathbf{D}$, denoted $\mathcal{B}$, consists of analytic functions f on $\mathbf{D}$ such that

$$\sup\{(1-|z|^2)|f'(z)| : z \in \mathbf{D}\} < +\infty.$$

Functions in the Bloch space have been studied extensively by many authors. See [**1**] for a recent survey of the theory of Bloch functions.

In this paper we study a class of generalized Bloch spaces. Specifically, for each $\alpha > 0$, we let $\mathcal{B}_\alpha$ denote the space of analytic functions f on $\mathbf{D}$ satisfying

$$\sup\{(1-|z|^2)^\alpha |f'(z)| : z \in \mathbf{D}\} < +\infty.$$

These spaces are not new. They are a certain type of Besov space [**12**]. When $\alpha > 1$, the space $\mathcal{B}_\alpha$ can be identified with the space of analytic functions f with

$$\sup\{(1-|z|^2)^{\alpha-1} |f(z)| : z \in \mathbf{D}\} < +\infty;$$

see Proposition 7. Such spaces are studied in [**13**] and [**16**]. When $0 < \alpha < 1$, the space $\mathcal{B}_\alpha$ can be identified with the analytic Lipschitz space $\mathrm{Lip}_{1-\alpha}$ consisting of analytic functions f on $\mathbf{D}$ such that

$$|f(z) - f(w)| \le C|z-w|^{1-\alpha}$$

for some constant $C > 0$ (depending on f) and all $z, w \in \mathbf{D}$; see [**21**] or Theorem B of [**10**]. Thus our results here unify the theory of Bloch functions, Lipschitz functions, and functions studied in [**13**] and [**16**].

Although most results in the paper are new (in the sense that this is the first time they appear in the literature), this paper is expository

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