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A CHARACTERIZATION OF WEIGHTED COMPOSITION OPERATORS

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ABSTRACT. Let V be a system of weights on a completely regular Hausdorff space X, and let E be a Hausdorff locally convex topological vector space. Then $CV_b(X,E)$ and $CV_0(X,E)$ are weighted spaces of vector-valued continuous functions on X topologized by the family of seminorms $f \to \sup\{v(x)p(f(x)) : x \in X\}$, where $v \in V$ and p is a continuous seminorm on E. In this note, we characterize weighted composition operators wC_T on $CV_b(X,E)$ induced by operator-valued functions w on X and self maps T on X. Some concrete examples are presented to illustrate the theory.

1. Introduction. Let X denote a completely regular Hausdorff space, V a system of weights on X, and let E be a Hausdorff locally convex topological vector space over the field $\mathbf{K} \in \{\mathbf{R}, \mathbf{C}\}$. Then $B(E)$ is the locally convex space of all continuous linear transformations (operators) on E with the topology of uniform convergence on bounded subsets of E, and $CV_b(X,E)$ and $CV_0(X,E)$ are weighted locally convex spaces of E-valued continuous functions on X topologized by the family of semi-norms $f \to \sup\{v(x)p(f(x)) : x \in X\}$, where $v \in V$ and p is a continuous semi-norm on E. If w is a $B(E)$-valued function on X and T is a self map on X such that $w \cdot f \circ T$ belongs to $CV_b(X,E)$ (or $CV_0(X,E)$) whenever $f \in CV_b(X,E)$ (or $CV_0(X,E)$), then the map taking f to $w \cdot f \circ T$ is a linear transformation on $CV_b(X,E)$ (or $CV_0(X,E)$), where $w \cdot f \circ T$ is defined as $(w \cdot f \circ T)(x) = w(x)(f(T(x)))$ for every $x \in X$. If this linear transformation is also continuous, we call it the weighted composition operator on $CV_b(X,E)$ (or $CV_0(X,E)$) induced by the pair (w,T) and denote it by the symbol wC_T. If $w(x) = I$, the identity transformation on E, for every $x \in X$, we write wC_T as C_T and call it the composition operator on $CV_b(X,E)$

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