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K-THEORY AND EXT-THEORY FOR RECTANGULAR UNITARY C^*-ALGEBRAS

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1. Introduction. Much study has been done on the C^*-algebras O_n generated by n isometries $S_1, S_2, \dots, S_n$ such that $S_1S_1^* + \cdots + S_nS_n^* = 1$. These algebras were introduced by Cuntz in [**9**] (see also [**6, 7, 8, 11, 15, 16**]). The K-theory of these algebras has been computed by Cuntz in [**7**]. The Ext-groups have been computed by Pimsner and Popa in [**16**] (see also [**15**]). In [**3**], Brown introduced the C^*-algebra U_n^{nc} generated by elements u_{ij}, $1 \leq i, j \leq n$, satisfying the relations which make the matrix $[u_{ij}]$ a unitary matrix. The K-groups of U_n^{nc} were computed in [**14**], where it was also shown that U_n^{nc} has no nontrivial projections. In [**18**], Voiculescu defined the $m \times n$ version of U_n^{nc} which we will denote $U_{(m,n)}^{\mathrm{nc}}$. The algebras O_n and U_n^{nc} correspond to $U_{(1,n)}^{\mathrm{nc}}$ and $U_{(n,n)}^{\mathrm{nc}}$, respectively. We will show that $U_{(m,n)}^{\mathrm{nc}}$ is isomorphic to the commutant of the $m+n$ by $m+n$ matrices in a certain amalgamated free product C^*-algebra. We will also prove some partial results about the K-theory of $U_{(m,n)}^{\mathrm{nc}}$ and also compute their Ext-groups.

2. The C^*-algebra $U_{(m,n)}^{\mathrm{nc}}$. We define $U_{(m,n)}^{\mathrm{nc}}$ as follows. $U_{(m,n)}^{\mathrm{nc}}$ is generated by elements u_{ij}, $1 \leq i \leq m$, $1 \leq j \leq n$, subject to the following relations on $u = [u_{ij}]$: $u^*u = I_n$ and $uu^* = I_m$, where I_k denotes the k by k identity matrix. $U_{(m,n)}^{\mathrm{nc}}$ has the universal property that if B is any unital C^*-algebra with elements v_{ij} for which $v = [v_{ij}]$ satisfies the same relations as u, then there is a unique unital $*$-homomorphism $\phi : U_{(m,n)}^{\mathrm{nc}} \to B$ such that $\phi(u_{ij}) = v_{ij}$. Clearly, any two C^*-algebras which satisfy the above property are canonically isomorphic. If u_{ij} and v_{kl} denote the generators of $U_{(m,n)}^{\mathrm{nc}}$ and $U_{(n,m)}^{\mathrm{nc}}$, respectively, then the map $u_{ij} \mapsto v_{ji}^*$ induces an isomorphism from $U_{(m,n)}^{\mathrm{nc}}$ onto $U_{(n,m)}^{\mathrm{nc}}$. As a result of this observation, we will restrict our attention to the $m \leq n$ cases.

There are two special cases of interest. If $m = n$, then $U_{(n,n)}^{\mathrm{nc}}$ is the C^*-algebra U_n^{nc} defined by Brown in [**4**]. If $m = 1$, then let $S_j = u_{1j}$.

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