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PRIME SUBMODULES OF NOETHERIAN MODULES

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0. Introduction. Let R be a ring. A proper left ideal L of R is prime if, for any elements a and b in R such that $aRb \subseteq L$, either $a \in L$ or $b \in L$. For example, any prime two-sided ideal is a prime left ideal. Prime left ideals have properties reminiscent of prime ideals in commutative rings. For example, Michler [**13**] and Koh [**7**] proved that the ring R is left Noetherian if and only if every prime left ideal is finitely generated. Moreover, Smith [**14**] showed that if R is left Noetherian (or even if R has left Krull dimension) then a left R-module M is injective if and only if, for every essential prime left ideal L of R and homomorphism $\varphi : L \to M$, there exists a homomorphism $\theta : R \to M$ such that $\theta|_L = \varphi$.

Several authors have extended the notion of prime left ideals to modules (see, for example, [**2, 3, 4, 6, 8, 9, 10, 11**]; in particular, [**3**] has a good bibliography). In this paper, we continue these investigations both in some generality and also in case M is a Noetherian module.

Let M be a left R-module. Then a proper submodule N of M is prime if, for any $r \in R$ and $m \in M$ such that $rRm \subseteq N$, either $rM \subseteq N$ or $m \in N$. It is easy to show that if N is a prime submodule of M then the annihilator P of the module M/N is a two-sided prime ideal of R. We consider which prime ideals P of R are the annihilators of modules M/N with N prime in M. A special class of prime submodules of M are the strongly prime submodules. Let K be a proper submodule of M, and let Q denote the annihilator of M/K. Then K is called strongly prime if (i) Q is a prime ideal of R and the ring R/Q is a left Goldie ring, and (ii) M/K is a torsion-free left (R/Q)-module. We investigate which prime ideals Q arise in this way.

We also are interested in chain conditions on (strongly) prime submodules of M. It is shown that if R satisfies the ascending chain condition (respectively, descending chain condition) on prime ideals then any finitely generated left R-module M satisfies the ascending chain

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