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SELF-SIMILARITY

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ABSTRACT. A self-similar set is defined as a compact set which is the union of its images under the members of a collection of contractions, the contractions being indexed by a compact set. Self-similarity is characterized by the consideration of points in the self-similar set as limits associated with certain sequences of contractions. Conditions are given for the occurrence of self-similarity. A self-similar set is also treated as a fixed point in hyperspace, and the continuous variation of self-similar sets is shown.

Introduction. Self-similarity has received much attention in recent years in connection with the study of fractals. The small-scale geometry of a self-similar set reproduces after a fashion the large-scale geometry of the set, such as seen in the Cantor space. The self-similar set is made up of arbitrarily small copies, or contracted images, of itself. Historically, Mandelbrot [**6**, p. 18] stated that many of his fractals were self-similar, and, in 1981, Hutchinson [**4**] put on a formal footing the matter of self-similarity in fractals. In 1985, the contribution of Hata [**3**] appeared. Also in 1985, Barnsley and Demko [**2**] formalized an approach to self-similar fractals. Barnsley [**1**] further expanded this approach in a text on fractals in 1988. In the present study the work of Hutchinson and Barnsley related to self-similarity is elaborated upon and expanded. Most studies of self-similarity have dealt with just finitely many contractions acting at once on a set, although Hata [**3**] does consider a sequence of contractions. In the present study the collection of contractions acting on a set may be uncountable.

In Section 1 "self-similar" is defined, and in Section 2 self-similarity is characterized by "addressing," or the consideration of points in the self-similar set as limits associated with certain sequences of contractions. The existence of a self-similar set determined by a certain type of collection of contractions is shown in Section 3. In Section 4 the

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