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STRONGLY EXTREME POINTS IN KÖTHE-BOCHNER SPACES

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ABSTRACT. The Kadec-Klee property with respect to a measure is discussed. A characterization of strongly extreme points of the unit sphere in certain Köthe-Bochner spaces is given.

1. Introduction. Let (Ω, Σ, μ) denote a measure space with σ-finite and complete measure μ and $L^0 = L^0(\Omega)$ denote the space of all (equivalence classes of) Σ-measurable real-valued functions, equipped with the topology of convergence in measure on μ-finite sets. In what follows, if $x, y \in L^0$, then $x \leq y$ means $x(t) \leq y(t)$ μ-almost everywhere in Ω.

For any Banach space X we denote by S_X the unit sphere of X.

A Banach subspace E of L^0 is said to be a *Köthe function* space (over (Ω, Σ, μ)) if

(i) $|x| \leq |y|$, $x \in L^0$, $y \in E$ imply $x \in E$ and $||x|| \leq ||y||$,

(ii) $\operatorname{supp} E := \cup\{\operatorname{supp} x : x \in E\} = \Omega$, where $\operatorname{supp} x = \{t \in \Omega : x(t) \neq 0\}$.

A Köthe function space E is said to be *order continuous* (respectively, *monotone complete*) provided $x_n \downarrow 0$ implies $||x_n|| \to 0$ (respectively $0 \leq x_n \uparrow x$, $x \in E$ imply $||x_n|| \to ||x||$).

Let E be a Köthe function space on (Ω, Σ, μ), X a Banach space. By $E(X)$ we denote the Banach space of all (equivalence classes of) strongly measurable functions $f : \Omega \to X$ such that $\bar{f} = ||f(\cdot)||_X \in E$ equipped with the norm $||f|| = ||\bar{f}||_E$.

Let E be a Köthe function space over (Ω, Σ, μ). E is said to have the (*positive*) *Kadec-Klee property* with respect to the measure μ (simply property (H_μ^+), respectively, (H_μ)), whenever $(x_n \stackrel{\mu}{\to} x, x_n, x \in E^+)$ $x_n \stackrel{\mu}{\to} x$ and $||x_n|| \to ||x||$ imply $x_n \to x$ strongly. Here

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