ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 23, Number 3, Summer 1993

NEW PROOFS OF BING'S 1-ULC TAMING THEOREM AND BING'S SIDE APPROXIMATION THEOREM

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The main contents of this paper are new, shorter proofs of R.H. Bing's 1-ULC taming theorem [**5**] and Side Approximation theorem [**6**]. Most of this paper is drawn from the author's dissertation, which he completed under the direction of Professors William T. Eaton and Michael P. Starbird of The University of Texas at Austin. The author thanks them for their patience in teaching and guiding him and thanks Matthew V. Brahm and James W. Cannon for suggestions that have proven helpful in writing this paper. Ideas and theorems of James W. Cannon are used frequently in this paper, so the reader is urged to consult [**12**].

Theorem (Bing). *Suppose that* Σ *is a* 2*-sphere topologically embedded in* E^3 *and that* $\operatorname{Int}\Sigma$ *is* 1*-ULC. Then* $\Sigma \cup \operatorname{Int}\Sigma$ *is a* 3*-cell.*

Since the new proof makes no use of Bing's approximability-implies-tameness theorem [**4, 12**, pp. 361–362], the latter theorem follows as a corollary from his 1-ULC taming theorem.

Theorem (Bing). *Suppose that* Σ *is a* 2*-sphere topologically embedded in* E^3 *and that, for each* $\varepsilon > 0$*, there is an* ε*-homeomorphism from* Σ *into* $\operatorname{Int}\Sigma$*. Then* $\Sigma \cup \operatorname{Int}\Sigma$ *is a* 3*-cell.*

The proof of Bing's 1-ULC taming theorem follows from Lemma 1 and the tools of [**12**, pp. 373–376]. No proof of Lemma 1 will be given, as its proof is easier than and uses the same methods as the proof of Lemma 2.

Lemma 1. *Suppose that* Σ *is a* 2*-sphere topologically embedded in*

Received by the editors on November 12, 1989, and in revised form on May 20, 1991.