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QUASI-UNIFORM STRUCTURES IN LINEAR LATTICES

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ABSTRACT. The study of some quasi-uniform structures likely to be defined in the usual normed lattices R^n, l_1, l_2 and $C([0,1])$, prompted a generalization of this particular method to the class of all normed lattices, in such a way that every normed linear lattice can be decomposed into two quasi-pseudometric linear structures (quasi-normed spaces), which enable us to restrict in some aspects the study of such normed lattices to its positive cone. All linear spaces under consideration are assumed to be defined over the field of real numbers.

1. Introduction. The three sources of material listed at the end may be useful to clearly illustrate the purpose of this paper. The idea of comparing ordered structures with uniform ones is at least as old as L. Nachbin's now classical *Topology and order*, [**3**]. We intend to use some results and terminology of linear lattice theory as it is exposed in G.J.O. Jameson's *Topology and normed spaces*, [**2**], as well as the topological ordered space concepts that can be seen in P. Fletcher and W.F. Lindgren's *Quasi-uniform spaces*, [**1**] in order to show that every normed lattice is determined by a quasi-uniform structure compatible with the linear and order structures of the space.

The following definitions can be found in [**2**, p. 375], except for the notion of E-space which is introduced here for the first time.

Definition 1.1. A normed lattice $(E, ||\quad||, \leq)$ is said to be an L-space, M-space or E-space, provided it satisfies that, for positive x, y:

$$||x+y|| = ||x|| + ||y||, \qquad ||x \vee y|| = ||x|| \vee ||y||,$$

or

$$||x+y||^2 + ||x-y||^2 = 2||x||^2 + 2||y||^2,$$

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