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POLYHEDRAL NORMS IN AN INFINITE DIMENSIONAL SPACE

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ABSTRACT. In finite dimensional linear spaces, polyhedral norms have been widely studied. Many extensions of such notions to infinite-dimensional spaces are possible: in fact, several different definitions have been given, leading to different classes of spaces; the comparison among these classes has not been studied in detail.

In the present paper we prove equivalences and inclusions among the classes considered in this context, and we indicate some counterexamples.

1. Introduction. In a finite dimensional linear space over the real field, a polytope is the convex hull of a finite set of points, or equivalently, the intersection of a finite number of closed half spaces. A finite dimensional normed space is said to be polyhedral if its unit ball is a polytope; note that such a space is polyhedral if and only if its dual space is also polyhedral.

In an infinite dimensional normed space X, we may consider several properties concerning the unit ball of X or of its dual space, which reduce to polyhedrality when the dimension of X is finite. In this paper we study eight properties of this kind, which have been introduced in the literature, and we establish relations among them. We study in depth two among the more important ones (polyhedrality according to Klee and quasi-polyhedrality according to Amir and Deutsch); several equivalent formulations of them are given. The particular case of Lindenstrauss spaces (spaces whose dual is linearly isometric to $L_1(\mu)$ for some measure μ) leads to a simpler situation.

2. Preliminaries. Throughout the paper, X is a normed space over $\mathbf{R}$, and unless otherwise stated, X is assumed to be infinite dimensional; B is its closed unit ball and S its unit sphere. The space X will always

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