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CONJUGACY CRITERIA FOR SECOND ORDER DIFFERENTIAL EQUATIONS

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ABSTRACT. Oscillation properties of linear differential equations of the second order

$$(*) \qquad (r(x)y')' + p(x)y = 0,$$

$x \in I = (a, b)$, $-\infty \le a < b \le \infty$, are viewed as a perturbation of the disconjugate equation

$$(**) \qquad (r(x)y')' = 0.$$

Sufficient conditions on the coefficients $r(x), p(x)$ ensuring that $(*)$ possesses a nontrivial solution having at least two zeros on I are obtained. It is shown that conjugacy criteria for $(*)$ are different in the case where the principal solutions y_a, y_b of $(**)$ at a and b are linearly independent or linearly dependent.

1. Introduction. In the present paper we deal with the second order differential equation of the form

$$(1.1) \qquad (r(x)y')' + p(x)y = 0,$$

where $x \in I = (a, b)$, $-\infty \le a < b \le \infty$, $r \in C^1(I)$, $r(x) > 0$ for $x \in I$.

Recall that two points x_1, $x_2 \in I$ are said to be conjugate relative to (1.1) if there exists a nontrivial solution y of this equation for which $y(x_1) = 0 = y(x_2)$. Equation (1.1) is said to be conjugate on I whenever there exists at least one pair of points of I which is conjugate relative to (1.1); in the opposite case equation (1.1) is said to be disconjugate on I.

The problem of disconjugacy of (1.1) on a given interval has a long history and disconjugacy results are exhibited in any monograph on

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