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TOEPLITZ OPERATORS ON THE DISK WITH LOCALLY SECTORIAL SYMBOLS

ALBRECHT BÖTTCHER

1. Introduction. Let $\mathbf{D}$ denote the open unit disk in $\mathbf{C}$, and let $A^2(\mathbf{D})$ be the Bergman space of square-integrable holomorphic functions in $\mathbf{D}$. For $a \in L^\infty := L^\infty(\mathbf{D})$, the Toeplitz operator $T(a)$ on $A^2(\mathbf{D})$ is defined by $T(a)\varphi = P(a\varphi)$ $(\varphi \in A^2(\mathbf{D}))$, where P is the orthogonal projection of $L^2(\mathbf{D})$ onto $A^2(\mathbf{D})$. The function a is usually referred to as the symbol of the operator $T(a)$.

The Fredholm properties of Toeplitz operators on $A^2(\mathbf{D})$ were studied by Venugopalkrishna [**15**] and Coburn [**7**] for symbols in $C(\bar{\mathbf{D}})$ (the functions continuous on the closed disk $\bar{\mathbf{D}}$), by McDonald [**10**] (and also in [**8**]) for symbols in $C(\bar{\mathbf{D}}) + H^\infty(\mathbf{D})$, and by McDonald and Sundberg [**12**] for symbols in $\operatorname{alg} \mathcal{H}L^\infty(\mathbf{T})$, the smallest closed subalgebra of L^∞ containing the bounded harmonic functions. Note that all these symbol classes are subalgebras of the algebra BC of all bounded continuous functions on $\mathbf{D}$.

Symbols which are not in BC were considered by Luecking [**9**] and McDonald [**11**]. Luecking established an invertibility criterion for $T(a)$ in case $a \geq 0$ a.e. on $\mathbf{D}$. McDonald proved a Fredholm criterion for $T(a)$ in the case where a belongs to $HC(\mathbf{D})$, the set of all functions $a \in L^\infty$ with the following property: for each $\tau \in \mathbf{T} := \partial\mathbf{D}$ there exists a set $U_\tau := \{z \in \mathbf{D} : |z - \tau| < \varepsilon\}$ and a straight line containing τ and dividing U_τ into two subsets U_τ^- and U_τ^+ such that $a|U_\tau^-$ and $a|U_\tau^+$ are uniformly continuous.

We remark that a major part of the aforementioned papers actually deal with Toeplitz operators on the ball $\{z \in \mathbf{C}^n : |z| < 1\}$ or on more general (and even exotic) domains. In addition to the works already cited, we refer in this connection to [**1**] and [**3**].

The present note concentrates on Toeplitz operators whose symbols are locally sectorial in some sense. Assume, for example, λ, μ, ν are

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