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ON GENERALIZED ABSOLUTE CESÀRO SUMMABILITY FACTORS

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ABSTRACT. In this paper using δ-quasi-monotone sequences a theorem on $|C,\alpha|_k$, $0<\alpha\le 1$, summability factors of infinite series, which generalizes a theorem of Mazhar [**6**] on $|C,1|_k$ summability factors, has been proved.

1. Introduction. A sequence (b_n) of positive numbers is said to be quasi-monotone if $n\Delta b_n \ge -\beta b_n$ for some β and is said to be δ-quasi-monotone, if $b_n \to 0$, $b_n > 0$ ultimately and $\Delta b_n \ge -\delta_n$, where (δ_n) is a sequence of positive numbers (see [**1**]). Let $\sum a_n$ be a given infinite series with partial sums (s_n). We denote by u_n^α and t_n^α the n-th Cesàro means of order α, $\alpha > -1$, of the sequences (s_n) and (na_n), respectively, i.e.,

$$u_n^\alpha = \frac{1}{A_n^\alpha}\sum_{v=0}^{n} A_{n-v}^{\alpha-1} s_v \tag{1.1}$$

$$t_n^\alpha = \frac{1}{A_n^\alpha}\sum_{v=1}^{n} A_{n-v}^{\alpha-1} v a_v, \tag{1.2}$$

where

$$A_n^\alpha = \binom{n+\alpha}{n} = \mathcal{O}(n^\alpha), \qquad \alpha > -1, \quad A_0^\alpha = 1 \quad \text{and} \quad A_{-n}^\alpha = 0 \quad \text{for } n>0. \tag{1.3}$$

The series $\sum a_n$ is said to be summable $|C,\alpha|_k$, $k \ge 1$, if (see [**3**])

$$\sum_{n=1}^{\infty} n^{k-1}|u_n^\alpha - u_{n-1}^\alpha|^k < \infty. \tag{1.4}$$

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