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BIFURCATION OF SYNCHRONIZED PERIODIC SOLUTIONS IN SYSTEMS OF COUPLED OSCILLATORS II: GLOBAL BIFURCATION IN COUPLED PLANAR OSCILLATORS

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ABSTRACT. We continue the study of a class of differential equations that govern the evolution of indirectly coupled oscillators. In a previous paper we established the existence of synchronized periodic solutions for weak and strong coupling. In this paper we present an example that shows an interesting behavior of the solutions for intermediate coupling strength. We analyze a two-parameter family of branches of periodic solutions and show when a branch has Hopf bifurcation points and/or turning points. We also study the stability of the periodic solutions.

1. Introduction. Many problems in physics, chemistry and biology involve systems of ordinary differential equations that govern the evolution of oscillatory subunits coupled indirectly through a passive medium [**10**]. In this paper we study the following system of ordinary differential equations, in which the oscillators that govern the states of the uncoupled subunits are all identical.

$$(1)\qquad \begin{aligned} \frac{dx_i}{dt} &= f(x_i) + \delta P(x_0 - x_i), \qquad i = 1, \dots, N, \\ \frac{dx_0}{dt} &= \varepsilon\delta P\bigg(\frac{1}{N}\sum_{i=1}^{N} x_i - x_0\bigg). \end{aligned}$$

Here the variable x_0 represents the state of the coupling medium through which the subunits are coupled. P is an $n \times n$ constant matrix of permeability coefficients or conductances, and the parameters ε^{-1} and δ measure the relative capacity of the coupling medium and the coupling strength, respectively [**4, 5**]. In the absence of coupling, the

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