ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 23, Number 4, Fall 1993

ON THE SECOND EIGENVALUE AND THE SECOND EIGENFUNCTION OF A CLASS OF SELFADJOINT SECOND ORDER LINEAR SYSTEMS

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ABSTRACT. The multiplicity of the second eigenvalue and the sign properties of the components of the second eigenfunction of the second order linear system $y''+\lambda P(X)y = 0, y(-1/2) = y(1/2) = 0$, where $P(x)$ is a 2×2 positive definite matrix-valued function with positive entries, are investigated. Sufficient conditions for the nondegeneracy of the second eigenvalue and the constant sign property for the components of the second eigenfunction are obtained.

1. Introduction. Let $P(x)$ be a continuous real $n \times n$ positive definite matrix-valued function defined on the interval $a \leq x \leq b$. Suppose $P(x) = (p_{ij}(x))$, $p_{ij}(x) > 0$ for $x \in [-1/2, 1/2]$. It is interesting to consider the following eigenvalue problem

$$\begin{aligned} y''(x) + \lambda P(x)y(x) = 0, \\ y(-1/2) = y(1/2) = 0, \end{aligned} \tag{1.1}$$

where $y(x)$ is an $\mathbf{R}^n$-valued function. The eigenvalue problem (1.1) is much more complicated than the classical scalar Sturm-Liouville eigenvalue problem. For the scalar Sturm-Liouville problem, all eigenvalues are nondegenerate, and the number of nodal points of the n^{th} eigenfunction is related to n in a clear way. But these scalar results are no longer true for (1.1) except for the first eigenvalue λ_1. In [**1,2,3**] S. Ahmad and A.C. Lazer proved some interesting results which tell us that under the assumption on the coefficient matrix $P(x)$ the first eigenvalue λ_1 of (1.1) is nondegenerate, and the components of the first

Received by the editors on November 11, 1991, and in revised form on October 21, 1992.
Key words and phrases. Selfadjoint second order linear system, eigenvalue, eigenfunction, matrix, characteristic value.
AMS *Subject Classifications.* 34A30, 34B25.
This research was supported in part by the National Science Council of the Republic of China.