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AUTOMORPHISMS OF THE INTEGRAL GROUP RING OF THE WREATH PRODUCT OF A p-GROUP WITH S_n, THE CASE $n = 2$

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1. Introduction. Let $\mathbf{Z}G$ be the integral group ring of the group G which is the wreath product $H \mathrm{wr} S_n$ where H is a finite p-group. It has been proved in [**1**] that if $n \geq 3$, then any normalized automorphism θ of $\mathbf{Z}G$ can be written as $\theta = \tau_u \circ \lambda$ where λ is an automorphism of G and τ_u is the inner automorphism of $\mathbf{Q}G$ induced by a suitable unit u of $\mathbf{Q}G$. We complete this work by proving the same result for $n = 2$. We use the notations of [**1**] and state the

Theorem. *Let G be the wreath product $H \mathrm{wr} S_2$ of a finite p-group H and S_2. Then every normalized automorphism θ of $\mathbf{Z}G$ can be written as $\theta = \tau_u \circ \lambda$ where λ is an automorphism of G and u is a unit of $\mathbf{Q}G$.*

2. Some observations. The group in question is

$$G = (H \times H) \rtimes \langle (12) \rangle = \{(a, b; \sigma) \mid a, b \in H, \sigma = (12) \text{ or } I\},$$
$$H \text{ a finite } p\text{-group.}$$

Identifying $(a, b; I)$ with (a, b) we have $(a, b)^{(12)} = (b, a)$. Denote by C_g the class sum of g and by $\mathcal{C}_g$ the class of g. We note that

$$\mathcal{C}_{(a,b)} = \{(a^x, b^y) \mid x, y \in H\} \cup \{(b^y, a^x) \mid x, y \in H\}.$$

Assume throughout that θ is a given normalized automorphism of $\mathbf{Z}G$. If $p = 2$, then G is a 2-group and the result is true by the Theorem of Weiss [**5**]. Thus we may assume that $p \neq 2$. Therefore, $\theta(\Delta(G, P)) = \Delta(G, P)$ where $P = H \times H$. We recall two crucial lemmas.

Lemma 1. *If $\theta(C_g) = C_x$, $\theta(C_h) = C_y$, then there exist $t, z \in G$ such that $\theta(C_{gh}) = C_{xy^t} = C_{x^z y}$.*

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