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BLOW-UP BEHAVIOR FOR SEMILINEAR HEAT EQUATIONS: MULTI-DIMENSIONAL CASE

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ABSTRACT. This paper is concerned with the Cauchy problem:

$$u_t - \Delta u = F(u), \qquad (x,t) \in R^N \times (0,T)$$
$$u(x,0) = u_0(x)$$

where $u_0(x)$ is continuous, nonnegative and bounded, and $F(u) = u^p$ with $p > 1$ or $F(u) = e^u$. Assume that u blows up at $x = 0$ and $t = T$. In case $F(u) = u^p$, let $w(y,s) = (T-t)^{1/(p-1)}u(y(T-t)^{1/2},t)$, $s = -\log(T-t)$. We study the large time behavior of $w(y,s)$. In the radial case, we prove: if $w(y,s) \not\equiv \beta^\beta$ ($\beta = (p-1)^{-1}$), then either $w(y,s) = \beta^\beta(1-(2ps)^{-1}NH(y)) + o(1/s)$ where $H(y) = (2N)^{-1}|y|^2 - 1$ or there exists an $m \geq 3$, $k_m > 1$, constants C_i (not all zero) and polynomials $H_{m,i}$ of degree m, such that $w(y,s) = \beta^\beta(1 - e^{(1-m/2)s}\sum_{i=1}^{k_m} C_i H_{m,i}(y)) + o(e^{(1-m/2)s})$. The above convergence takes place in C^2_{loc} as well as in some weighted Sobolev space. For the nonradial solutions, we also obtain some results in the case $N = 2$. Similar results also hold in the case $F(u) = e^u$.

1. Introduction. This paper is concerned with nonnegative blowing up solutions of the initial value problem:

$$u_t = \Delta u + F(u) \quad \text{in} \quad R^N \times (0,T) \tag{1.1}$$

$$u(x,0) = u_0(x), \qquad x \in R^N \tag{1.2}$$

where $u_0(x)$ is continuous, nonnegative and bounded, and

$$F(u) = u^p \quad \text{with} \quad p > 1, \qquad \text{or} \quad F(u) = e^u. \tag{1.3}$$

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