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EXTENDED SPECTRAL RADIUS IN TOPOLOGICAL ALGEBRAS

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0. Introduction. In this note we discuss several definitions of spectral radius in locally convex topological algebras as introduced by Zelazko [**5**]. In particular, we present what is known about the equivalence or nonequivalence of these definitions in various classes of algebras. Some of these relations were already proven by Zelazko [**5**], some others are more recent [**1, 2, 3**], and some are new.

Spectral radius is one of the most important features of the Banach algebras theory. For an x in a commutative, complex Banach algebra $\mathcal{A}$ with unit e, we have many equivalent definitions:

$$R(x) = \lim_n \sqrt[n]{||x^n||},$$

$$R(x) = \sup\{|\lambda| : x - \lambda e \text{ is not invertible}\},$$

$$R(x) = \sup\{|F(x)| : F \text{ is a linear and multiplicative functional}\},$$

$$R(x) = \inf\left\{r : \text{ if the radius of convergence of } \sum_{n=0}^{\infty} \alpha_n \lambda^n \text{ is } r \right.$$
$$\left. \text{then } \sum_{n=0}^{\infty} \alpha_n x^n \text{ converges in } \mathcal{A}\right\}$$

and a number of others. Most of these definitions can be used to extend the notation of spectral radius to more general topological algebras; however, the extended definitions are no longer equivalent. In the next sections we discuss the relations between them.

1. Definitions and notation. A locally convex algebra $\mathcal{A}$ is a topological Hausdorff algebra which is a locally convex space. The topology on such an algebra can be introduced by a family $\{||\cdot||_\alpha\}$, $\alpha \in \Lambda$, of seminorms such that, for any α there is a β such that

$$||xy||_\alpha \leq ||x||_\beta ||y||_\beta, \tag{1}$$

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