ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 24, Number 1, Winter 1994

AN EPIDEMIOLOGICAL MODEL WITH POPULATION SIZE DEPENDENT INCIDENCE

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Dedicated to Paul Waltman on the occasion of his 60th birthday

ABSTRACT. An SIS epidemiological model with a general population-size dependent disease incidence, reduced fertility and vertical transmission is analyzed. Complete global stability analyses and explicit threshold parameters are given. The fractions in the disease states usually approach a disease-free or endemic equilibrium while the population size grows to infinity, decays to zero or approaches a finite size. In one case there is a center. The persistence of the disease and disease-related deaths affects the behavior of the population size.

1. Introduction. Consider the spread of an infectious disease in a host population with exponential births and deaths defined as follows. Let $N(t)$ be the total population size as a function of time t, b be the natural birth rate constant, and d be the natural death rate constant. Then, assuming that the natural births and the natural deaths are proportional to the total population size, we have

$$N'(t) = (b-d)N, \qquad N(0) = N_0 \tag{1.1}$$

as the initial value problem. The net growth rate is therefore $r = b-d$ so that the population size $N(t)$ grows exponentially if $r > 0$, is constant if $r = 0$ and decays exponentially if $r < 0$.

Assume that the rate at which new individuals become infected (the incidence rate) is proportional to number Y of infectives, times the population-size dependent contact rate $\lambda(N)$, times the probability of contacting a susceptible individual, X/N, where X is the number of susceptible individuals. Thus the incidence is $\lambda(N)YX/N$. The contact rate $\lambda(N)$ is the daily average number of contacts of an infective which are sufficient for transmission if the person contacted is susceptible

Received by the editors on March 9, 1993.
Key words. Epidemiological model, global stability, nonlinear incidence rate, thresholds, varying population size.